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Monitoring the Monitor: An Incentive Structure for a Financial Intermediary

Stefan Krasa
Anne P. Villamil

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Monitoring the Monitor: An Incentive Structure for a Financial Intermediary

Stefan Krasa
Department of Economics
University of Illinois
University of Vienna

Anne P. Villamil
Department of Economics
University of Illinois

ABSTRACT

The purpose of this paper is to extend the bilateral contracting problem studied in Townsend (1979) and Gale and Hellwig (1985) to an explicit, multilateral contracting arrangement which resembles banking. We derive the structure of an optimal contract for a large but finite intermediary, establish gains from delegated monitoring, and study the incentive problem of the monitor in an economy where the intermediary has no information, risk, or cost advantage relative to individual agents. Unlike previous delegated monitoring studies, the *law of large numbers* is not sufficient to obtain our results. Instead, we appeal to a stronger result, the *large deviation principle*, which establishes that convergence in the law of large numbers is exponential.

1 Introduction

The literatures on both contract theory and on the nature of financial intermediaries have grown extensively in recent years. However, much of the first literature has restricted attention to bilateral agreements between representative agents. For example, Townsend (1979) and Gale and Hellwig (1985) consider single representative investor-entrepreneur pairings in markets with information asymmetries where agents must trade to facilitate production. This restriction to bilateral representative agent contracts is appropriate in the Townsend and Gale and Hellwig studies since the central research question addressed is the allocative effect of information problems in competitive economies. In contrast, recent contributions to the financial intermediation literature have extended bilateral contract models to multilateral settings to generate intermediary-like institutions endogenously. For example, Diamond (1984) and Boyd and Prescott (1986) explain the existence of financial intermediaries as cost-minimizing institutional responses to the need for information production. Boyd and Prescott use the theory of mechanism design to endogenously motivate intermediation, while Diamond establishes that it may be optimal for investors to delegate monitoring tasks to an intermediary.

In this paper we study the delegated monitoring problem posed by Diamond in a multilateral version of the Gale and Hellwig model. We consider an economy with a large number of investors and entrepreneurs who must trade to facilitate production. Entrepreneurs have private information about their output, but a costly technology exists which can be used to verify returns ex-post. Agents have the option to either contract directly or to elect an intermediary to represent their interests. In the first case, each investor must monitor each project that he/she has an interest in independently. This is the one-sided contract problem studied by Gale and Hellwig. In the second case, investors elect a representative to whom they delegate the monitoring task (the "intermediary" or "delegated monitor"). This intermediary con-

tracts with all remaining investors and each entrepreneur, and is charged with the responsibility of monitoring the entrepreneurs. This two-sided contract problem is the main focus of our paper.

The crucial question which arises in a delegated monitoring setting is—who monitors the monitor? Two previous mechanisms have been proposed to ensure that the intermediary truthfully reports actual payoffs to the investors. First, Diamond (1984, p. 369) assumes that the intermediary is subject to unbounded, non-pecuniary penalties such as lost time and reputation, or less plausibly, physical punishment. Unfortunately, the unbounded and imprecise nature of this enforcement mechanism is difficult to rationalize on economic grounds. Further, as Gale and Hellwig (1985, p. 649) note: The imposition of these penalties is "equivalent to perfect bond-posting," and consequently the incentive problem "becomes innocuous." Second, Williamson (1986, p. 169) assumes that the intermediary's actual monitoring costs are fixed and *independent of its size*. This assumption implies that the intermediary's marginal monitoring costs decrease as the size of the intermediary increases. Unfortunately, this specification confers on the intermediary an inherent cost advantage (i.e., a natural monitoring monopoly), and hence the dominance of delegated monitoring over direct investment is not surprising.[1]

We study an alternative incentive mechanism, costly state verification of both the entrepreneurs and the intermediary, in an economy where the intermediary has *no* information, risk, or cost advantage vis-a-vis investors. We show that under this mechanism, delegated monitoring strictly dominates direct investment and simple debt is an optimal contract for all agents. Previous delegated monitoring studies have established gains from intermediation by appealing to the *law of large numbers*.[2] However, in our symmetric

[1]Of course Williamson's main concern was credit rationing.

[2]Diamond shows that the penalties per entrepreneur go to zero as the number of entrepreneurs (I) becomes large. Williamson shows that the expected cost of monitoring the monitor goes to zero as I becomes large, *when the costs are independent of* I.

cost setting an investor's cost of monitoring the monitor is increasing in the number of entrepreneurs, and hence becomes infinite as the number of entrepreneurs goes to infinity. Clearly, the law of large numbers is not sufficient to establish gains from monitoring in this context. Instead, we use the *large deviation principle,* a stronger result, which shows that convergence in the law of large numbers is exponential. This alternative mathematical approach is essential in the economy that we consider.

The paper is organized as follows. Section 2 contains a description of the model. The main result from Section 3 establishes that there are gains from intermediation. In Section 4 we show that if there are sufficiently many entrepreneurs, two-sided simple debt is an optimal contract. Section 5 contains essential mathematical results, and Section 6 contains concluding remarks.

2 The Model

Consider an economy with finite numbers of two types of risk neutral agents, investors and entrepreneurs. Each entrepreneur $i = 1, \ldots, I$ is endowed with a risky investment project which transforms y units of a single input at time 0 into θ_i units of ouput at time 1, but has no endowment of the input. In contrast, each investor $j = 1, \ldots, J$ is endowed with 1 unit of the homogeneous input, but has no direct access to a productive technology. We assume that y is an integer with $y > 1$. Hence, the project of an entrepreneur cannot be financed by a single investor.[3] The total available supply of investment is larger than the input required by all entrepreneurs, so J investors can be accomodated by the I entrepreneurs (i.e., $J = yI$). All entrepreneurs and investors are symmetrically informed about the distribution of θ_i at time 0, but asymmetric information exists about the state of the project's actual re-

[3]This assumption implies that there is duplicative monitoring in the absence of intermediation, because more than one investor must monitor a single entrepreneur. Delegated monitoring economizes on these costs (c.f., Diamond (1984)).

alization ex-post. In particular, only entrepreneur i freely observes the actual realization of θ_i at time 1, where this actual realization is denoted by w.

As in Townsend (1979) suppose that a technology exists which can be used to verify w to non-owners at time 1. Let this verification technology have the following characteristics:

(T1) Use of the state verification technology is costly; and

(T2) w is privately revealed only to the individual who requests (deterministic) costly state verification (CSV).[4]

Assumption (T1) is similar with the specification of the CSV technology in Townsend, except that as in Gale and Hellwig (1985, p. 651) the verification cost is comprised of both a pecuniary component and an indirect "pecuniary equivalent" of a non-pecuniary cost. The non-pecuniary costs permit negative utility but rule out negative consumption, while pecuniary equivalents of non-pecuniary costs ensure that the costs can be shared by the contracting parties.[5] In contrast, assumption (T2) differs fundamentally from the specification in Townsend. In his model w is *publicly* announced after CSV occurs, while in our model w is *privately* revealed only to the agent who requests CSV. Assumption (T2) is essential for our analysis since if all information could be made public ex-post, there would be no need to monitor the monitor. However, it also appears to accurately describe the privacy and institutional features which characterize most lending arrangements.[6]

The following assumptions summarize technical aspects of the economy.

(A1) The θ_i are independent, identically distributed random variables on the probability space $(\Omega, \mathcal{A}, P)$.[7]

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[4]Stochastic verification is discussed in the concluding section.

[5]These costs may be thought of as the money paid to an attorney to file a claim (a pecuniary cost) and the monetary value of time lost when visiting the attorney (a pecuniary equivalent). Note that Diamond's costs were unbounded while these costs are fixed.

[6]Diamond (1984, p. 395) observes: "Financial intermediaries in the world monitor much information about their borrowers in enforcing loan covenents, but typically do not directly announce this information or serve an auditor's function."

[7]We will always refer to this probability space without mentioning it explicitly, when

(A2) The distribution F has a continuously differentiable density f with respect to the Lebesgue measure, and $f(x) > 0$ for every $x \in [0, T]$.

(A3) The ex-post verification cost is a fixed constant.

Because entrepreneurs have technologies but no input and investors have input but no technologies, it is clear that agents will wish to trade in this economy to facilitate production. The remainder of this section will be devoted to specifying contracts which govern trade among agents and the optimization problems from which these contracts can be derived. Formal proofs that the contracts are indeed optimal are deferred to Sections 3 and 4.

2.1 The Direct Investment Problem

Let all direct, bilateral interactions between investors and entrepreneurs be regulated by a contract whose general form is defined as follows.

Definition 1. A one-sided contract between an investor and an entrepreneur is a pair $(R(\cdot), S)$, where $R(\cdot)$ is an integrable, positive payment function on $\mathbb{R}_+$, such that $R(w) \leq w$ for every $w \in \mathbb{R}_+$ and S is an open subset of $\mathbb{R}_+$ which determines the state where monitoring occurs.

As is standard practice in this literature, we restrict the universe of contracts that we consider to the set of incentive compatible contracts. Let $C = (R(.), S)$ denote this set. The following condition ensures that all contracts under consideration satisfy this restriction. There exists $\bar{R} \in \mathbb{R}_+$ such that $S = \{w: R(w) < \bar{R}\}$. It is well known that the imposition of this restriction is without loss of generality because any arbitrary contract can be replaced by an incentive compatible contract with the same actual payoff (c.f., Townsend (1979, p. 270)). Therefore, the set of all incentive compatible contracts is fully specified by the tuple $(R(\cdot), \bar{R})$.

writing P for probability or E for expected value.

We will study a particular type of contract, called a simple debt contract (SDC), which we describe in Definition 2. The fact that simple debt is the optimal contract among all one-sided investment schemes is a well known result which we state formally in Theorem 1.

Definition 2. $(R(\cdot), \bar{R})$ is a simple debt contract if: $R(w) = w$ for $w \in S = \{w < \bar{R}\}$ and $R(w) = \bar{R}$ if $w \in S^c = \{w \geq \bar{R}\}$.

The payment schedules in Definition 2 resemble simple debt because:

(i) When verification does not occur the payment to the investor is constant (i.e., the entrepreneur pays a fixed amount $\bar{R}$ for all realizations of the state above some cutoff level), where S^c is the complement of S.

(ii) When verification does occur the payment to the investor is state contingent (i.e., the entrepreneur pays the entire realization for all outcomes below the cutoff), where S is viewed as the set of bankruptcy states.

We will often denote SDCs by $\bar{R}$. These contracts have been studied extensively by Townsend (1979), Gale and Hellwig (1985), and others. It is well known that SDCs arise as optimal (cost-minimizing) responses to asymmetric information problems in economies with costly deterministic state verification technologies. In essence, agents minimize verification costs in such economies by verifying only low realizations of θ_i and accepting fixed payments (which do not require monitoring) in all other states.

The investor and the entrepreneur's direct investment problem can now be stated formally:

$$\max_{(R(\cdot),S)\in C} \int_0^T [w - R(w)]\, dF(w)$$

subject to

$$\frac{I}{J} \int_0^T R(w)\, dF(w) - \int_S c\, dF(w) \geq r. \tag{1}$$

This "one-sided problem" describes the nature of trade when investors and entrepreneurs contract directly: The expected utility of a representative

entrepreneur is maximized, subject to a constraint that the expected return of a representative investor, net of monitoring costs (c), be at least as great as some reservation level (r). Note that $R(\cdot)$ is the total payment to all investors, and I/J is the number of investors required to fund a single project. The problem reflects the assumption that credit markets are competitive.[8]

2.2 The Delegated Monitoring Problem

We will now construct an alternative, multilateral contract problem that we will refer to as the intermediary's "two-sided problem." The essential difference between the two problems is as follows. When investors and entrepreneurs write direct bilateral contracts, each investor must monitor each entrepreneur with whom he/she contracts in certain states of nature, so duplicative monitoring is inherent in this setting. In contrast, if investors elect a monitor to perform the verification task, it may be possible to eliminate some of the duplicative monitoring (even though investors must monitor the monitor in some states of nature).

Consider now the delegated monitoring problem (i.e., the election of an intermediary). The lending market is competitive, hence it follows that an investor who wishes to act as an intermediary must offer contracts which maximize the expected utility of the entrepreneurs and assure each investor of at least the reservation level of utility. Otherwise, agents would trade directly or another intermediary would offer an alternative contract (i.e., there is free entry into intermediation) with terms that are preferable to the I entrepreneurs and/or the remaining $J-1$ investors. Let $(R(\cdot), S)$ denote aspects of the two-sided contract which pertain to the entrepreneur-intermediary relationship and $(R^*(\cdot), S^*)$ denote aspects of the two-sided contract which

[8]We consider an economy in which there are more agents who wish to invest than investment opportunities. Since the supply of loans is inelastic, the level of return necessary to attract investors is driven down to the reservation level r.

pertain to the intermediary-investor relationship.[9] Thus, the intermediary's problem embodies optimization by all agents in the economy.

Before stating the intermediary's problem, we must first derive the random variables which describe the income from its portfolio. Recall that $R(w)$ denotes the payoff by an entrepreneur to the intermediary if output w is realized and θ_i is the random variable which describes the output w of a particular entrepreneur i in state ω. Consequently, the intermediary's income from this entrepreneur, given transfer $R(\cdot)$, is $G_i(R(\cdot);\omega) = R(\theta_i(\omega))$. The random variables G_i are independent for each choice of $R(\cdot)$ because the θ_i are independent. Assume that the intermediary contracts with $i = 1, 2, \ldots, I$ entrepreneurs. Thus, the average income of the intermediary per entrepreneur under payment schedule $R(\cdot)$, denoted by $G^I(R(\cdot);\omega)$, is:

$$G^I(R(\cdot);\omega) = \frac{1}{I}\sum_{i=1}^{I} G_i(R(\cdot);\omega). \tag{2}$$

Finally, let $F^I(\cdot)$ be the distribution function of $G^I(\cdot)$.

The purpose of this paper is to establish gains from delegated monitoring in an economy where the intermediary has no information, risk, or cost advantage vis-a-vis investors. Since we do not distinguish the intermediary a priori, it has no information advantage. Further, since investors are risk neutral it has no risk advantage. The investors' cost structure for monitoring the monitor remains to be specified. Let c denote the *actual* fixed cost incurred by the intermediary when it monitors entrepreneur i, and c_I^* denote the *actual* cost incurred by an investor when he/she monitors the intermediary with portfolio of size I. The *expected* monitoring costs are of primary importance to the intermediary and the investors when they make their decisions at time 0 (i.e., $\int_S c\, dF(\cdot)$ and $\int_{S^*} c_I^*\, dF^I(\cdot)$ respectively). These *expected*

[9]Recall that $(R(.), S)$ is also used to denote the entrepreneur-investor contract in the one-sided model. We avoid introducing additional notation at this point because the incentive problem associated with ensuring that entrepreneurs report truthfully is the same regardless of whether they report to the investors directly or to the intermediary.

costs depend on three factors: the actual costs (c and c_I^*), the relevant states (S and S^*), and the size of the intermediary (via c_I^* and $F^I(\cdot)$).

It is clearly not reasonable to assume that the intermediary's actual monitoring costs are independent of its size. Although cost and other asymmetries may exist in actual economies (providing other reasons for the existence of intermediaries), we wish to evaluate the efficacy of delegated monitoring as an independent rationale for intermediation. Hence we make the following *cost symmetry* assumption:

(CS) The costs of monitoring the monitor c_I^* are linearly increasing in its size.[10]

Assumption CS is fulfilled, for example, when state verification by investors involves verifying the full state (i.e., each of the I projects in the intermediary's portfolio). Our goal in this paper is to establish gains from monitoring which stem solely from the intermediary's ability to eliminate duplicative (but symmetric) monitoring costs. If investors can economize further on these costs (e.g., by monitoring only insolvent firms or monitoring stochastically), then delegated monitoring will be even more attractive. Thus, we view this example as a benchmark case.[11]

The two-sided contract between the intermediary and each entrepreneur, and the intermediary and the investors, can now be defined.

Definition 3. A two-sided contract is a four-tuple $((R(\cdot), S), (R^*(\cdot), S^*))$ having the following properties:

(i) $R(\cdot)$ is an integrable positive payment function from an entrepreneur to the intermediary such that $R(w) \leq w$ for every $w \in \mathbb{R}_+$, and S is an

[10] If the intermediary's actual cost is bounded from above its marginal cost of contracting with additional firms is decreasing in I at a rate faster than $1/I$, giving it an inherent cost advantage relative to individual investors.

[11] Many symmetric cost structures satisfy assumption CS, hence it is not restrictive. We discuss this example because if we can establish gains from intermediation for a benchmark case (i.e., when the institutional structure is such that intermediary failures are quite costly) then clearly our arguments will hold for less costly cases. Exponentially increasing costs are also permissible, as long as they do not increase faster than the bound given by equation (21). However, we regard such exponentially increasing costs as implausible.

open subset of $\mathbb{R}_+$ which determines the set of all realizations of an entrepreneur's project where the intermediary must monitor;

(ii) $R^*(\cdot)$ is an integrable positive payment function from the intermediary to the investors such that $R^*(w) \leq w$. For every realization w of $G^I(\cdot)$,[12] the payment to an individual investor is given by $\frac{I}{J-1}R^*(w)$;[13] and S^* is an open subset of $\mathbb{R}_+$ which determines the set of all realizations of the intermediary's income from the entrepreneurs the investors must monitor.

We now derive the set of all incentive compatible two-sided contracts. As in the one-sided problem, we again restrict our analysis to this set without loss of generality. Clearly each entrepreneur will again announce an output which minimizes its payment obligations to the intermediary. Let $\bar{w} = \arg\min_{x \in S} R(x)$ be the output that minimizes this payoff over all non-monitoring states, and recall that w is observed directly in the monitoring states S. Consequently, the announcement by an entrepreneur is given by $\arg\min_{x \in \{w, \bar{w}\}} R(x)$. A similar condition holds for the intermediary-investor portion of the contract (i.e., $R^*(\cdot), \bar{R}^*$). As in the one-sided problem, the following condition ensures that all contracts are incentive compatible. There exist $\bar{R}, \bar{R}^* \in \mathbb{R}_+$ such that $S = \{w: R(w) < \bar{R}\}$ and $S^* = \{w: R^*(w) < \bar{R}^*\}$. The set of all incentive compatible two-sided contracts is fully specified by the four-tuple $(R(\cdot), \bar{R}), (R^*(\cdot), \bar{R}^*)$.

Finally, a two-sided simple debt contract can now be defined:

Definition 4. A contract $(R(\cdot), \bar{R})$, $(R^*(\cdot), \bar{R}^*)$ is a two-sided simple debt contract (denoted by $(\bar{R}, \bar{R}^*)$) if:

(i) $R(w) = w$ for $w \in S = \{w < \bar{R}\}$ and $R(w) = \bar{R}$ if $w \in S^c = \{w \geq \bar{R}\}$; and

[12] Recall that $G^I(\cdot)$ is the average income per entrepreneur defined by equation (2).

[13] It is convenient to define $R^*(\cdot)$ as the total payment by the intermediary to investors per entrepreneur. Therefore, in order to derive the payment to an individual investor we must multiply this amount with $\frac{I}{J-1}$.

(ii) $R^*(w) = w$ for $w \in S^* = \{w < \bar{R}^*\}$ and $R^*(w) = \bar{R}^*$ if $w \in S^{*c} = \{w \geq \bar{R}^*\}$.

We will often denote two-sided simple debt contracts by $(\bar{R}, \bar{R}^*)$.

The intermediary's two-sided optimization problem can now be stated:

$$\max_{(R(\cdot),\bar{R}),(R^*(\cdot),\bar{R}^*)} \int_0^T [w - R(w)]\, dF(w)$$

subject to

$$\frac{I}{J-1} \int_0^T R^*(w)\, dF^I(R(\cdot), \bar{R})(w) - \int_{S^*} c_I^*\, dF^I(R(\cdot), \bar{R})(w) \geq r \qquad (3)$$

$$I \left[\int_0^T [w - R^*(w)]\, dF^I(R(\cdot), \bar{R})(w) - \int_S c\, dF(w) \right] \geq r. \qquad (4)$$

This problem states that the intermediary maximizes the expected utility of each ex-ante identical entrepreneur subject to two constraints. (3) states that the expected payoff to the $J-1$ remaining investors (i.e., those who did not become intermediaries) must be at least r, the reservation level of utility. (4) states that the profit from intermediation (i.e., net payoffs from the entrepreneurs less the payoff to the investors) must also be at least r.

3 Delegated Monitoring

In this section we obtain two main results. First, Theorem 1 establishes that simple debt is the optimal contract among all one-sided investment schemes. This is the Gale and Hellwig (1985) result for our economy. Second, Theorem 2 establishes that two-sided simple debt with delegated monitoring dominates one-sided direct investment if there are sufficiently many entrepreneurs. The proof relies on the *large deviation principle*. This principle, and its relation to the *law of large numbers*, is also discussed. We prove the optimality of two-sided simple debt contracts in Section 4. Throughout these sections we appeal to certain mathematical results proved in Section 5.

Theorem 1. *Simple Debt is the optimal contract among all one-sided investment schemes.*

Theorem 1 is proved in Gale and Hellwig (1985) as Proposition 4. A similar result is also proved in Williamson (1986). The strategy of the proof is as follows. Consider two optimal contracts: Let $\bar{R}$ be a simple debt contract and $(A(\cdot), \bar{A})$ be some alternative contract. Since both contracts are optimal, both must yield the same expected payoff to entrepreneurs. With the first contract investors request verification if $w < \bar{R}$. In the alternative contract, verification occurs in all states w such that $w < \bar{A}$. Since $\bar{A} > \bar{R}$ (otherwise the contracts cannot have the same return to entrepreneurs) the expected verification costs must be less for the simple debt contract $\bar{R}$.

Theorem 2. *Two-sided simple debt contracts with delegated monitoring strictly dominate one-sided direct investment if there are sufficiently many entrepreneurs.*

Proof. Our arguments depend on the continuity of the constraints, established by Lemma 1 in Section 5, and can be summarized as follows. Let $\bar{R}$ be the simple debt contract which is optimal among all one-sided schemes described by Theorem 1. We show that there exists an $\bar{R}^*$ such that (3) is binding for the two-sided contract $(\bar{R}, \bar{R}^*)$, and that by increasing $\bar{R}^*$ the payoff to investors increases and (3) does not bind.[14] We next show that (4) is fulfilled but not binding. Hence increasing $\bar{R}^*$ slightly makes both constraints slack if the number of entrepreneurs is sufficiently large. This proves the Theorem since by lowering $\bar{R}$ we can make the entrepreneurs better off than in the one-sided scheme.

We begin by showing that the costs of monitoring the monitor go to zero if $\bar{R}^*$ is less than the intermediary's expected return from one entrepreneur,

[14]In general, the investors' payoff does not increase monotonically with $\bar{R}^*$ because the probability that investors must monitor the intermediary is an increasing function of $\bar{R}^*$. This is also true for one-sided schemes (c.f., Gale and Hellwig (1985, p. 662)).

$\bar{R}^* < E[G_i(R(\cdot))]$, and if the intermediary is sufficently large. This means:

$$\lim_{I\to\infty} \int c_I^* \, dF^I(w) = 0 \quad \text{for every } \bar{R}^* < E[G_i(R(\cdot))]. \tag{5}$$

(5) follows immediately from (21) of Lemma 4 in Section 5, which establishes that the probability of a default by the delegated monitor (i.e., the probability that an investor must monitor the monitor), converges to zero exponentially. However, by assumption (CS) the monitoring costs c_I^* increase only linearly. The key insight of the proof is that, in expected terms, the costs go to zero.

The Theorem is proved as follows:

(i) The *law of large numbers* implies that

$$\lim_{I\to\infty} \int R^*(w) \, dF^I(R(\cdot))(w) = \bar{R}^* \quad \text{for every} \quad \bar{R}^* < E[G_i(R(\cdot))]. \tag{6}$$

(6) indicates that the probability of a default by the delegated monitor goes to zero. Hence investors get the face value of the SDC with certainty.

(ii) (5) and (6) imply that we can find a two-sided contract $(\bar{R}, \bar{A}^*)$ such that (3) is fulfilled for sufficiently large I but not binding. Because of the continuity of (3) with respect to $\bar{R}$ and $\bar{A}^*$ (Lemma 1), there must exist a face value $\bar{R}^* < \bar{A}^*$ such that (3) is binding for the two-sided contract $(\bar{R}, \bar{R}^*)$. By construction, increasing $\bar{R}^*$ slightly implies that the investors' payoff increases.

(iii) All that remains to show is that (4) is also fulfilled, but not binding. Because of (5), it follows that $\int_S c \, dF > \int_S c_I^* \, dF^I$ for all sufficiently large I. This and the fact that (3) is binding implies

$$I \int_0^T R^*(w) \, dF^I < (J-1)(r + \int_S c \, dF). \tag{7}$$

Consequently,

$$I \left[\int_0^T [w - R^*(w)] \, dF^I - \int_S c \, dF \right]$$
$$> I E[G_i(\bar{R})] - J \int_S c \, dF - (J-1)r \geq Jr - (J-1)r = r,$$

where the first inequality follows from (7) and the second inequality follows from the fact that $\bar{R}$ must fulfill constraint (1) of the one-sided problem by assumption. This proves Theorem 2.

In the proof of Theorem 2 we use the *large deviation principle.* This principle is related to the *law of large numbers* but is stronger. In particular, the large deviation principle shows that convergence in the law of large numbers is exponential. The principle is essential in establishing equation (5) in the proof, and is referenced formally in the proof of Lemma 4 in Section 5. The economic intuition behind the problem addressed by Theorem 2 is that monitoring costs are linearly increasing in the number of entrepreneurs (i.e., as the size of the intermediary gets large, the verification costs, c_I^*, in the the bankruptcy state become large as well). However, if the probability of default goes to zero "fast enough," then the expected value of the costs of monitoring the monitor become insignificant for a well diversified intermediary – even though the intermediary is of finite size and hence is not *perfectly* diversified. The role of the *large deviation principle* is to provide a convergence result that is "fast enough" (i.e., faster than that provided by the law of large numbers) to generate gains from intermediation.

Theorem 2 establishes that two-sided arrangements are better than one-sided direct investment. However, the following problem remains: If (3) and (4) are not binding it is not clear who gets the surplus. Thus, the maximization problem is not well defined. The following proposition shows that this difficulty does not arise and that optimal contracts exist.

Proposition 1. *If there are sufficiently many entrepreneurs, then there exist optimal contracts among the set of all two-sided simple debt contracts. The two constraints from the intermediary's optimization problem bind for all optimal contracts.*

Proof. The result follows directly from the continuity results of Lemma 1 in

Section 5. The existence of optimal contracts is straightforward since according to Lemma 1 both the constraints and the argument we are maximizing over are continuous functions of $\bar{R}$ and $\bar{R}^*$. To show that both constraints must bind for optimal contracts, consider the following cases.

(i) Suppose by way of contradiction that at an optimum both constraints do not bind. Then $\bar{R}$ can be reduced slightly without violating the constraints. This, however, contradicts the optimality of the contract, so it is not possible that both constraints are slack at an optimum.

(ii) Suppose that only (3) does not bind. Then $\bar{R}^*$ can be reduced slightly without violating the constraint. This reduces the total payment of the intermediary to the investors, but (4) will no longer bind and we can apply the above argument to get a contradiction.

(iii) Finally, suppose that only (4) does not bind. We must show that (3) no longer binds if $\bar{R}^*$ is increased by a small amount. This is not straightforward since by increasing $\bar{R}^*$ we automatically increase the expected cost of monitoring the monitor. However, (6) establishes that this expected monitoring cost goes to zero as the size of the intermediary increases. Further, it follows from (4) that $\bar{R}^*$ remains bounded away from $EG_i(\bar{R})$ as I increases.[15] Thus, the expected cost of monitoring the monitor remains close to zero (i.e., changes very little) if we increase $\bar{R}^*$ slightly. Consequently, if only (4) does not bind the gain from a higher payment to investors exceeds the loss from an increase in monitoring expenditures and (3) does not bind, a contradiction. Hence, both constraints must bind at an optimum.

4 Optimality of Two-Sided Simple Debt

The proof that two-sided simple debt is optimal requires a slightly stronger result than that used in the previous section. In particular, in the Proof of

[15] Divide both sides of (4) by I and take the limit for $I \to \infty$.

Theorem 2 we used Lemma 4 from Section 5 to establish that the probabilities of default converged exponentially to zero. We now use Lemma 5 from Section 5, to show that the densities also converge exponentially to zero. From this result we get the following Corollary which establishes that the difference in probability between the realization being below x_1 and x_2, respectively, is bounded by the absolute value of the difference between x_1 and x_2 times a term which converges exponentially to zero.

Corollary 1. *Let $\bar{\bar{R}} > 0$ and $z < EG_i(\bar{\bar{R}})$. Then there exist $a > 0$ and $\bar{I} > 0$ such that $|P(G^I(\bar{R}) \leq x_1) - P(G^I(\bar{R}) \leq x_2)| \leq e^{-aI}|x_1 - x_2|$ for every x_1, $x_2 \leq z$, for every $I \geq \bar{I}$, and for every $\bar{R} \geq \bar{\bar{R}}$.*

This Corollary is essential for establishing the main Theorem of this Section. The main problem of the proof is that two-sided simple debt contracts do not necessarily minimize the expected costs of monitoring the monitor, as the following example shows:

Example 1. In order to simplify the computations, we consider a discrete distribution. Using simple approximation arguments it is easy to extend the example to the case of continuous distributions. Assume that there are two entrepreneurs $i = 1, 2$, and that the realization of θ_i is 0 with probability 0.4; and 1 and 2 each with probability 0.3. Let $(\bar{R}, \bar{R}^*)$ be a simple debt contract with $\bar{R} = 1$ and $\bar{R}^* = 0.7$;[16] and $(A(\cdot), \bar{R}^*)$ be an alternative contract such that $A(0) = A(1) = 0$ and $A(2) = 2$. The investor-intermediary part of the contract is the same in both cases, and both contracts yield the same expected return to the entrepreneurs. However, the probability of a default by the intermediary is lower with the second contract. Specifically, it is 0.49 for the alternative contract and 0.64 for the simple debt contract.

As we have seen in Theorem 1, simple debt contracts are optimal for the one-sided problem. Any other contracts generate higher expected costs

[16]Recall that $\bar{R}^*$ is the total payment by the intermediary to the investors *per firm*.

for monitoring the entrepreneurs. In the two-sided case we essentially must minimize the sum of the expected costs of monitoring the entrepreneurs and of monitoring the intermediary. Unfortunately, the second summand need not be minimal for two-sided simple debt as Example 1 shows. The main idea of the proof of the following Theorem is to show that the one-sided and two-sided problems are essentially the same for large intermediaries (i.e., changes in the intermediary-entrepreneur part of the contract have a very small effect on investors' payoffs because of Corollary 1. Consequently minimizing the expected costs of monitoring the entrepreneurs is the main concern).

Theorem 3. *If there are sufficiently many entrepreneurs then the optimal contracts are two-sided simple debt contracts. Two-sided simple debt contracts strictly dominate all other types of contracts.*

Proof. We proceed by contradiction. Assume without loss of generality, that there exists some alternative two-sided contract $(A_I(\cdot), A_I^*(\cdot))$, for every I, which improves upon the optimal two-sided simple debt contract of Theorem 2. By Theorem 2, we can restrict our analysis to two-sided contracts. We show:

(i) The investor's part of contract $A_I^*(\cdot)$ must be a simple debt contract, $\bar{A}_I^*$.

Next we choose a two-sided simple debt contract $(\bar{R}_I, \bar{R}_I^*)$ such that entrepreneurs have the same expected return and the expected payments from the intermediary to the investors remain constant.[17] Furthermore, from Lemma 3 in Section 5, $\bar{A}_I^* \geq \bar{R}_I^*$.[18] The contracts $(\bar{R}_I, \bar{R}_I^*)$ must fulfill the conditions of Corollary 1 for all sufficiently large I (i.e., there exists $\bar{\bar{R}} > 0$ and $z < EG^I(\bar{\bar{R}})$ such that $\bar{R}_I^* \leq z < EG^I(\bar{\bar{R}}) \leq \bar{R}_I$ for all sufficiently large I).[19] We show:

[17]First choose $\bar{R}_I$ such that $EG^I(\bar{R}_I) = EG^I(A_I(\cdot))$. Because of the continuity results of Lemma 1, a SDC, $\bar{R}_I^*$, can be chosen such that $\int R_I^*(w)\, dF(R(\cdot)) = \int A_I^*(w)\, dF(A(\cdot))$.

[18]This holds since by Lemma 3, $\int A_I^*(w)\, dF(R(\cdot)) \geq \int A_I^*(w)\, dF(A(\cdot))$. Thus, to get equality we must choose $\bar{R}_I^* \leq \bar{A}_I^*$.

[19]By the (indirect) assumption of the proof, $(A_I(\cdot), A_I^*(\cdot))$ dominates the optimal two-sided SDCs of Theorem 2. This is only possible if the probability that investors must mon-

(ii) Using $(\bar{R}_I, \bar{R}_I^*)$ instead of $(A_I(\cdot), \bar{A}_I^*)$, the left hand side of (3) decreases by at most $c_I^* e^{-aI} |\bar{A}_I - \bar{R}_I|$.[20] The left hand side of (4) increases by at least $Icm|\bar{A}_I - \bar{R}_I|$, where $m = \min_{x \in [0,T]} f(x)$,[21] because with simple debt contracts the intermediary must monitor entrepreneurs in fewer states of nature. This is essentially the two-sided analogue of Theorem 1.

(iii) Since for large I the surplus is much greater than the loss, we can show that it is possible to distribute some of the intermediary's gain to the investors by increasing the face value of $\bar{R}_I^*$ such that both constraints are satisfied and *not* binding. Hence, the face value of $\bar{R}_I$ can be lowered such that both constraints still hold. The entrepreneurs are better off with a contract with a lower face value. Consequently, the two-sided simple debt contract $(\bar{R}_I, \bar{R}_I^*)$ dominates $(A_I(\cdot), \bar{A}_I^*)$, which provides the contradiction. Therefore all optimal contracts must be simple debt contracts.

We now prove claim (i). This follows immediately from Theorem 1 because the intermediary is like an entrepreneur whose production is described by the random variable $G^I(\cdot)$.

Next we prove claim (ii). In order to compute the change of the left hand side of (3) we need only compute the change in expected monitoring costs (because the first integral on the left-hand side of (3) does not change by construction of $(\bar{R}, \bar{R}^*)$). In the following, let $\bar{A}_I$ be the simple debt contract

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itor the intermediary goes to zero as I gets large. Otherwise the expected monitoring costs would go to infinity. Thus, in the limit investors receive the face value $\bar{A}_I^*$ with certainty, i.e., $\lim_{I\to\infty} \int A_I^*(w)\, dF(A(\cdot)) = \lim_{I\to\infty} \bar{A}_I^*$. Clearly, the costs of monitoring an individual entrepreneur $\int_S c\, dF(w)$ remain bounded away from zero as $I \to \infty$. Dividing both sides of (4) by I and taking the limit we conclude that $\lim_{I\to\infty} \bar{A}_I^* < \lim_{I\to\infty} EG^I(A(\cdot))$, so there exist z, z' such that $\bar{A}_I^* \leq z < z' \leq EG^I(A(\cdot))$ for all sufficiently large I. Since $\bar{R}_I^* \leq \bar{A}_I^*$ by Lemma 3 (c.f., previous footnote) and $EG^I(R_I(\cdot)) = EG^I(A_I(\cdot))$ by definition, it follows that $\bar{R}_I^* \leq z < z' \leq EG^I(R_I(\cdot))$. Now choose $\bar{\bar{R}}$ such that $z < \bar{\bar{R}} \leq R_I$ for all sufficiently large I. This is exactly the condition of Corollary 1.

[20]The two-sided SDC does not necessarily minimize the probability of default by the intermediary. Thus the left hand side of (3) may be smaller under contract $(A_I(\cdot), \bar{A}_I^*)$.

[21]$m > 0$ by assumption (A2).

with the same face value as $A_I(\cdot)$. Observe that:

$$\int_{S^*_{\bar{R}^*_I}} c^*_I \, dF(\bar{R}_I) - \int_{S^*_{\bar{A}^*_I}} c^*_I \, dF(A_I(\cdot)) \leq \int_{S^*_{\bar{R}^*_I}} c^*_I \, dF(\bar{R}_I) - \int_{S^*_{\bar{R}^*_I}} c^*_I \, dF(\bar{A}_I). \quad (8)$$

This inequality follows from two factors: First, the income of the intermediary from contract $\bar{A}_I$ is higher than from contract $A_I(\cdot)$ in all states, hence less monitoring occurs; and second $\bar{A}^*_I \geq \bar{R}^*_I$.

Clearly, the difference in payoff from an individual entrepreneur to the intermediary between the two SDCs with face values $\bar{A}_I$ and $\bar{R}_I$ is at most $\bar{A}_I - \bar{R}_I$.[22] Therefore

$$G^I(\bar{R}_I) = \frac{1}{I}\sum_{i=1}^{I} G_i(\bar{R}_I) \geq \frac{1}{I}\sum_{i=1}^{I}[G_i(\bar{A}_I) - (\bar{A}_I - \bar{R}_I)] = G^I(\bar{A}) - (\bar{A}_I - \bar{R}_I). \quad (9)$$

Hence (9) implies,

$$P\{G^I(\bar{A}_I) \leq \bar{R}^*_I\} \geq P\{G^I(\bar{R}_I) \leq \bar{R}^*_I - (\bar{A}_I - \bar{R}_I)\}. \quad (10)$$

From (8), (10) and Corollary 1 it now follows that

$$\int_{S^*_{\bar{R}^*_I}} c^*_I \, dF(\bar{R}_I) - \int_{S^*_{\bar{A}^*_I}} c^*_I \, dF(A_I(\cdot)) \leq c^*_I e^{-aI}(\bar{A}_I - \bar{R}_I). \quad (11)$$

The intermediary's loss (i.e., the decrease of the left hand side of (4)) can be computed using the main idea of Theorem 1: If agents use contract $A_I(\cdot)$ instead of $\bar{R}_I$, the intermediary must monitor in additional states $w \in [\bar{R}_I, \bar{A}_I]$. Hence, expected monitoring costs increase by $\int_{\bar{R}_I}^{\bar{A}_I} c \, dF \geq cm(\bar{A}_I - \bar{R}_I)$, and the total loss is at least $Icm|\bar{A}_I - \bar{R}_I|$. This proves (ii).

Finally we prove (iii). Let $\varepsilon > 0$. From footnote 19 we have $R^*_I \leq z < EG^I(\bar{\bar{R}}) \leq \bar{R}_I$. Hence, by the law of large numbers there exist $\bar{h} > 0$ and $\bar{I}$ such that

$$P(\{G^I(\bar{R}_I) \geq \bar{R}^*_I + \bar{h}\}) > 1 - \varepsilon, \quad \text{for all } I \geq \bar{I}. \quad (12)$$

[22] $\bar{A}_I - \bar{R}_I > 0$ since both contracts have the same expected return but $\bar{R}_I$ is a SDC (hence it has the lowest face value among all contracts with the same expected return).

(12) and Corollary 1 imply that by increasing the face value of $\bar{R}_I^*$ by $h < \bar{h}$, the payoff to investors (i.e., the left hand side of (3)) increases by at least $\frac{I}{J-1}h(1-\varepsilon) - hc_I^* e^{-aI}$. If I is sufficiently large, this amount is bounded below by $\frac{I}{J}h(1-2\varepsilon)$. Again because of (12) the intermediary's profit (i.e., the left hand side of (4)) is decreased by at most $Ih(1-\varepsilon)$.

By choosing $h = \frac{J}{I}\frac{1}{1-2\varepsilon}c_I^* e^{-aI}(\bar{A}_I - \bar{R}_I)$ constraint (3) is fulfilled and not binding (by the computations in the previous paragraph). Given this h, the profit of the intermediary decreases by at most $I\frac{J}{I}\frac{1-\varepsilon}{1-2\varepsilon}c_I^* e^{-aI}(\bar{A}_I - \bar{R}_I)$. Comparing this to the total gain, which is Icm, it is clear that an $\bar{\bar{I}}$ can be chosen independently of $\bar{A}_I$, such that the gain is greater than the loss.[23] This means that constraint (4) is not binding as well, which proves the Theorem.

5 Proofs of Mathematical Results

The following notation will be useful in the analysis that follows. Given some two-sided contract, let $\Gamma_1(\cdot)$ denote the aggregate payoff of the $J-1$ investors from the intermediary who interacts with I entrepreneurs, and γ_1 denote the expected payoff of a single investor from the intermediary. For the two-sided SDC with delegated monitoring, $(R(\cdot), \bar{R}), (R^*(\cdot), \bar{R}^*)$ these payoffs are:

$$\Gamma_I((R(\cdot), \bar{R}), (R^*(\cdot), \bar{R}^*)) = \int_0^T R^*(w)\, dF^I(R(\cdot), \bar{R})(w); \tag{13}$$

$$\gamma_I((R(\cdot), \bar{R}), (R^*(\cdot), \bar{R}^*)) = \frac{I}{J-1}\left[\Gamma_I(\cdot)\right] - \int_S c^*\, dF^I(\cdot). \tag{14}$$

The following Lemma establishes that the above payoff functions are continuous in the face values $\bar{R}$ and $\bar{R}^*$ for SDCs. This proves continuity of constraint (3). We also prove results which are necessary to get continuity of constraint (4) and of the argument of the two-sided optimization problem.

[23] Clearly this is the case if $\frac{J}{I}\frac{1-\varepsilon}{1-2\varepsilon}c_I^* e^{-aI} < cm$ which holds for all sufficiently large I and is independent of $\bar{A}_I$ and $\bar{R}_I$.

Lemma 1. *Let $R(w)$ be one-sided simple debt with face value $\bar{R}$. Then the functions $\bar{R} \mapsto \int R(w)\,dF(w)$ and $\bar{R} \mapsto \int_{w<\bar{R}} c\,dF(w)$ are continuous. Furthermore, $(\bar{R}, \bar{R}^*) \mapsto \Gamma_I(\bar{R}, \bar{R}^*)$ is continuous; and $(\bar{R}, \bar{R}^*) \mapsto \gamma_I(\bar{R}, \bar{R}^*)$ is continuous at every $(\bar{R}, \bar{R}^*) \in \mathbb{R}^2$, such that $\bar{R}^* < \bar{R}$.*

Proof. The proof for the first two functions is straightforward. We now prove continuity of Γ_I. Note that,

$$P(\{G^I(\bar{R}) \geq \bar{R}^*\}) \leq P(\{G^I(\bar{R}+h) \geq \bar{R}^*\}) \leq P(\{G^I(\bar{R}) \geq \bar{R}^* - h\}), \quad (15)$$

for $h > 0$ because of (10). Therefore $\int f(x)\,dF^I(\bar{R}+h)(x) \leq \int f(x+h)\,dF(\bar{R})(x)$, for every increasing step-function f, and hence for every arbitrary increasing function f by a standard approximation argument. Further, $\|A(\cdot) - R(\cdot)\|_\infty \leq |\bar{A} - \bar{R}|$, for all simple debt contracts $\bar{A}$ and $\bar{R}$. Thus,

$$|\Gamma_I(\bar{R}+h, \bar{R}^*+h^*) - \Gamma_I(\bar{R}, \bar{R}^*)| \leq |h| + |h^*|, \quad (16)$$

for every $h, h^* > 0$ and hence for every $h, h^* \in \mathbb{R}$. This proves continuity of Γ_I. It now remains to prove that $(\bar{R}, \bar{R}^*) \mapsto \int_{-\infty}^{\bar{R}^*} c^*\,dF(\bar{R})$ is continuous at every $(\bar{R}, \bar{R}^*) \in \mathbb{R}$ such that $\bar{R}^* < \bar{R}$. The distribution of $G^I(\bar{R})$ has a density which is bounded by a $K_n \in \mathbb{R}$ in $(-\infty, R)$ (c.f. Lemma 4). Therefore (15) implies $|\int_{-\infty}^{\bar{R}^*} c^*\,dF^I(\bar{R}) - \int_{-\infty}^{\bar{R}^*} c^*\,dF^I(\bar{R}+h)| \leq K_n|h|$. Since this inequality holds for every $\bar{R}^* < \bar{R}$ and since $\bar{R}^* \mapsto \int_{-\infty}^{\bar{R}^*} c^*\,dF^I(\bar{R})$ is clearly continuous, this proves continuity of γ_I.

To prove optimality of two-sided simple debt we need an additional technical Lemma which shows that two-sided simple debt contracts maximize total payments to investors (not including monitoring costs). The result is formally stated in Lemma 3. It follows immediately from the next Lemma.

Lemma 2. *Let μ be a probability measure on $[0, M]$. Let $R(\cdot)$ and $A(\cdot)$ be two contracts with the same expected value. We assume that $R(\cdot)$ is a simple*

debt contract $\bar{R}$. Then the simple debt contract is less risky than $A(\cdot)$ in the sense of Rothschild and Stiglitz; i.e., $\int u(A(w))\,d\mu(w) \leq \int u(R(w))\,d\mu(w)$, for all concave functions u.

If we interpret u as a utility function, then Lemma 2 essentially says that all risk averse consumers prefer the simple debt contract to any other contract with the same expected value. This is one of the criteria for comparing the riskiness of two distribution of Rothschild and Stiglitz (1970). By using Theorem 2 of their paper which proves the equivalence of three different concepts, and by using their argument on p. 230 ff, the proof of the Lemma is straightforward. The idea to use the Rothschild and Stiglitz criterion for the proof of Lemma 3 is due to Martin Hellwig. We now give the proof.

Proof of Lemma 2. Let H_A and H_R be the cumulative density functions of the distributions of $A(\cdot)$ and $R(\cdot)$, respectively. Let $G(t) = H_A(t) - H_R(t)$, and let $T(y) = \int_0^y G(x)\,dx$. Then $A(\cdot)$ is more risky than $R(\cdot)$ if the following two conditions are fulfilled:

$$(17)\quad T(M) = 0, \qquad \text{and} \qquad (18)\quad T(y) \geq 0 \qquad \text{for every } y \in [0, M].$$

Let g be the density of G. Partial integration yields $T(M) = \int_0^M G(x)\,dx = xG(x)|_0^M - \int_0^M xg(x)\,dx = 0$, since the integral over $xg(x)$ is just the expected value of $A(\cdot) - R(\cdot)$. This proves (17). Now note that $G(t) \geq 0$ for every $0 \leq t < \bar{R}$, and that $G(t) \leq 0$ for every $\bar{R} \leq t \leq M$. This together with (17) proves (18). Theorem 2 of Rothschild and Stiglitz (1970) implies that $A(\cdot)$ is more risky than $R(\cdot)$. This proves the Lemma.

Lemma 3. *Let $(\bar{R}, \bar{R}^*)$ be a two-sided simple debt contract and $(A(\cdot), \bar{R}^*)$ be an alternative contract where $EG^I(A(\cdot)) = EG^I(\bar{R})$. Then $\Gamma_I(\bar{R}, \bar{R}^*) \geq \Gamma_I(A(\cdot), \bar{R}^*)$.*

Proof. Note that

$$\Gamma_I(R(\cdot), R^*(\cdot)) = \iint \cdots \int R^*(\frac{1}{I}\sum_{i=1}^{I} R(w_i))\, dF(w_1)dF(w_2)\ldots dF(w_I). \quad (19)$$

Since $R^*(\cdot)$ is a concave function we can apply Lemma 2 and inductively substitute $R(\cdot)$ by $A(\cdot)$ in (19). This proves the Lemma.

Next we show that convergence in the law of large number is exponential. This result is used in Section 3 to establish gains from delegated monitoring. The proof follows immediately from the large deviation principle.

Lemma 4. *Let $(X_i)_{i\in\mathbb{N}}$ be a sequence of independent identically distributed random variables with values in $[0, M]$ and distribution μ. Let μ_n be the distribution of $\frac{1}{n}\sum_{i=1}^{n} X_i$. Then $\mu_n([0, b])$ converges to zero exponentially as $n \to \infty$ for every $b < EX_i$.*

Proof. $M(\xi) = \int e^{\xi x}\, d\mu(x) < \infty$ for every $\xi \in \mathbb{R}$, since μ has a compact support. Let $\mathcal{I}(x) = \sup_{\xi\in\mathbb{R}}(\xi x - \log M(\xi))$. Then Cramérs theorem (Stroock p.30 ff.) implies that $\mathcal{I}$ is a "rate function", and that μ_n satisfies the *large deviation principle with rate* $\mathcal{I}$. Consequently

$$\limsup_{n\in\mathbb{N}} 1/n \log \mu_n([0, b]) \leq -\inf_{x\in[0,b]} \mathcal{I}(x), \quad (20)$$

for every $b \in \mathbb{R}$. We now prove that $\inf_{x\in[0,b]} \mathcal{I}(x) > 0$ for every $b < EX_i$:

$\mathcal{I}(x)$ is decreasing on $(-\infty, EX_i]$ (Stroock Lemma (3.3)). Therefore we need only prove that $\mathcal{I}(b) > 0$ for every $b < EX_i$. For fixed b let $H(\xi) = \xi b - \log M(\xi)$. Since $H(0) = 0$, it is sufficient to show that $H'(0) \neq 0$. This, however, can be easily verified:

$$H'(\xi) = b - \frac{\int x e^{\xi x} f(x)\, dx}{\int e^{\xi x} f(x)\, dx}$$

consequently $H'(0) = b - EX_i \neq 0$. This and (20) imply that for every $b < EX_i$ there exists an $a > 0$ and an integer $\bar{n}$ such that $1/n \log \mu_n([0, b]) \leq -a$

for every $n \geq \bar{n}$. Consequently

$$\mu_n([0,b]) \leq e^{-an} \qquad \text{for every } n \geq \bar{n}. \tag{21}$$

For the optimality of two-sided simple debt contracts proved in Section 4 we need a stronger result than Lemma 4. In particular, we must show that the densities of μ_n also converge to zero exponentially on every interval $[0,b]$ where $b < EX_i$.

Lemma 5. *Let $(X_i)_{i\in\mathbb{N}}$ be a sequence of independent, identically distributed random variables with values in $[0,M]$ and distribution μ. We assume that $\mu = \tilde{\mu} + \lambda\delta_R$ where $\operatorname{supp}\tilde{\mu} \subset [0,R]$ and $\tilde{\mu}$ has a density f with respect to the Lebesgue measure on $\mathbb{R}$. We assume that f is continuously differentiable on $[0,M]$. Let μ_n be the distribution of $\frac{1}{n}\sum_{i=1}^{n} X_i$, and let δ_R be the Dirac point measure. Then $\mu_n = \tilde{\mu}_n + \lambda_n\delta_R$ where $\operatorname{supp}\tilde{\mu}_n \subset [0,R]$. $\tilde{\mu}_n$ has a density f_n with respect to the Lebesgue measure which converges uniformly to zero as $n \to \infty$ on all compact subsets of $[0, EX_i)$. This convergence is exponential.*

The idea of the proof of Lemma 5 is as follows. We first show that the derivative of the density functions of $G^I(R(\cdot))$ is bounded by a polynomial term (24). This is straighforward if the densities are continuously differentiable with compact support in $\mathbb{R}$, see for example Floret (1981) Exercise 14.24. In our case the densities are discontinuous at 0 and R which requires us to deal with the derivatives at these two points separately. The idea of the proof, however, is essentially the same. If the Lemma does not hold then this immediately implies that the measure of an interval $[0,b]$ can only converge to zero with polynomial speed (25). However, by Lemma 4 the probability of every such interval with $b < G^I(R(\cdot))$ converges to zero with exponential speed as $I \to \infty$ (21). This contradiction proves Lemma 5.

Proof: We first derive an upper boundary for the derivative of f_n. The main problem is that f is discontinous at 0 and R as a function defined on $\mathbb{R}$.

CLAIM 1: Let u, v be functions on $\mathbb{R}$ with support in $[0,T]$. Let u be continuous from the right and bounded. Let v be continuously differentiable in $[0,T]$. Then the convolution $u * v$ is continuous from the right and $\|(u * v)'\|_\infty \leq \|u\|_1 \|v'\|_\infty + 2|u\|_\infty \|v\|_\infty$.

Let $z(t,x,h) = u(t)\frac{1}{h}[v(x+h-t)-v(x-t)]$. Furthermore let $B_{1,h} = \{t: -h \leq x-t \leq 0\}$ and let $B_{2,h} = \{t: T-h \leq t-x \leq T\}$. Then

$$\int_{B_{1,h}} z(t,x,h)\,dt = h\int_0^1 z(x+th,x,h)dt = \int_0^1 u(x+th)v(h-th)\,dt.$$

Consequently $\lim_{h\downarrow 0}\int_{B_{1,h}} z(t,x,h)\,dt = u(x)v(0)$, since u and v are differentiable from the right. A similar argument yields $\lim_{h\downarrow 0}\int_{B_{2,h}} z(t,x,h)\,dt = u(x)v(T)$. Therefore

$$(u*v)'(x) = \lim_{h\downarrow 0}\int z(t,x,h)\,dt = \int u(t)v'(x-t)\,dt + u(x)v(0) + u(x)v(T),$$

which proves claim 1.

CLAIM 2: The distribution of $\sum_{i=1}^n X_i$ has a density g_n on $[0,nR)$. The point $\{nR\}$ has probability λ^n. The right-hand derivative $g_n'(x)$ exists for every $n \in \mathbb{N}$ and for every $x \in [0,nR)$. Further, there exists a constant $K \leq 2\|f'\|_\infty + 4\|f\|_\infty^2$ such that $|g_n'(x)| \leq n^2K$ for every $n \in \mathbb{N}$ and for every $x \in [0,nR)$.

For the proof of claim 2 we proceed by induction. For $n = 1$ there is nothing to prove. We now assume that the inequality holds for $n-1$. Using the formula for the density of the sum of two independent random variables and accounting for the "point mass" λ at R and λ^{n-1} at $(n-1)R$ we get

$$g_n(x) = \int g_{n-1}(t)f(x-t)\,dt + \lambda^{n-1}f(x-(n-1)R) + \lambda g_{n-1}(x-R), \quad (22)$$

for every $x \in [0,nR]$. (22) implies

$$\|g_n\|_\infty \leq \|g_{n-1}\|_1\|f\|_\infty + \|f\|_\infty + \|g_{n-1}\|_\infty \leq 2\|f\|_\infty + \|g_{n-1}\|_\infty.$$

Consequently

$$\|g_n\|_\infty \leq 2n\|f\|_\infty. \tag{23}$$

Using (22), (23) and claim 1 yields

$$\|g_n'\|_\infty \leq \|g_{n-1}\|_1 \|f'\|_\infty + 4n\|f\|_\infty^2 + \lambda^{n-1}\|f'\|_\infty + \lambda\|g_{n-1}'\|_\infty.$$

By substituting the induction hypothesis for $K = 2\|f'\|_\infty + 4\|f\|_\infty^2$, we conclude the proof of claim 2.

Note that $f_n(x) = ng_n(nx)$ is the density of $\tilde{\mu}_n$ with respect to the Lebesgue measure. We get

$$|f_n'(x)| \leq n^4 K, \text{ for every } n \in I\!N, \text{ and for every } x \in [0, R\,). \tag{24}$$

In the second part of the proof we proceed indirectly. Assume that there exists a $b < EX_i$ such that f_n does not converge uniformly to zero on $[0, b]$ with exponential speed. That means that there exist a sequence ε_n that does not converge exponentially to zero and a sequence $(y_n)_{n \in I\!N}$ in $[0, b]$ such that $f_n(y_n) \geq \varepsilon$ for every $n \in I\!N$. We prove that under these circumstances, (24) implies that $\mu_n([0, b])$ cannot converge to zero exponentially which contradicts Lemma 4.

Let

$$h_n(x) = \begin{cases} \varepsilon - n^4 K(x - y_n), & \text{if } y_n \leq x \leq y_n + \frac{\varepsilon}{n^4 K}; \\ 0, & \text{otherwise.} \end{cases}$$

Let $b' \in (b, EX_i)$. Then there exists an integer $\bar{\bar{n}}$ such that $y_n + \frac{\varepsilon}{n^4 K} \leq b'$ for every $n \geq \bar{\bar{n}}$. Consequently $0 \leq h_n(x) \leq f_n(x)$ for every $n \geq \bar{\bar{n}}$, because of (24). Therefore

$$\mu_n([0, b']) = \int_0^{b'} f_n(x)\,dx \geq \int_0^{b'} h_n(x)\,dx = \frac{\varepsilon^2}{2n^4 K}. \tag{25}$$

By (21) and (25) we get $\frac{\varepsilon^2}{2n^4 K} \leq e^{-an}$ for every $n \geq \max(\bar{n}, \bar{\bar{n}})$, a contradiction. This proves the Lemma.

We are now ready to prove Corollary 1 from Section 4.

Proof of Corollary 1. The distribution of θ_i has by assumption a density f which is continuously differentiable on $[0, M]$. The densities of $F^1(\bar{R})$ are then given by $f_{\bar{R}} = f|_{[0,\bar{R}]} + \delta_{\bar{R}}$, so the constant K of (24) can be chosen independently of $\bar{R}$. Thus inequality (25) holds uniformly for every $\bar{R}$. Further, $P(G^n(\bar{R}) \leq x) \leq P(G^n(\bar{\bar{R}}) \leq x)$ for every $x \in I\!R$. Thus, (21) holds uniformly for every distribution μ_n of $G^n(\bar{R})$, where $\bar{R} \geq \bar{\bar{R}}$, so Lemma 5 also holds uniformly for every $\bar{R} \geq \bar{\bar{R}}$. This proves the Corollary.

6 Concluding Remarks

In this paper we extend the Gale and Hellwig (1985) contract problem to the delegated monitoring setting proposed by Diamond (1984). As in Diamond, a crucial problem in the analysis is to ensure that the monitor reports truthfully to investors. We use a two-sided version of Townsend's (1979) fixed cost, deterministic state verification technology to solve the monitor's incentive problem. We show that delegated monitoring dominates direct investment and that two-sided simple debt is optimal. Our economic environment requires us to introduce new mathematical arguments based on the large deviation principle. This mathematical technique is essential in our model because when the cost of monitoring the monitor depends on its size, the law of large numbers is not sufficient to establish gains from intermediation. In addition, even if monitoring costs are independent of size, the large deviation argument is necessary to derive the optimality of simple debt (since uniform convergence of the densities does not follow from the law of large numbers).

We obtain our results in an economy where the intermediary has no inherent information, risk, or cost advantage, and where monitoring is deterministic. Recently, costly state verification studies (c.f., Townsend (1988)

and Mookherjee and Png (1989)) have shown that the form of the optimal contract may be altered under stochastic monitoring. In contrast, our delegated monitoring result is not affected by stochastic monitoring. This follows from the fact that as in the deterministic case, the probability that a state occurs in the stochastic case which triggers monitoring goes to zero exponentially. Hence the expected cost of monitoring the monitor goes to zero as well, and delegated monitoring continues to dominate direct investment. Our rationale for studying deterministic monitoring is similar to that given for the cost symmetry assumption in Section 2. We wish to establish gains from delegated monitoring which stem solely from the intermediary's ability to eliminate duplicative verification costs for a benchmark case. If other factors exist which further reduce monitoring costs (e.g., stochastic monitoring), intermediation will be even more attractive.

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